Dynamic Centrality in Random Subnetworks
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Social networks are usually scale-free with well-connected hubs.

On any given day, only some links are active, and the daily networks can vary greatly. In particular, the hubs can change from day to day.

This is called Dynamic Centrality.

**Our Model**
We start with a scale-free underlay \( N_U \) nodes, that represents the overall set of relationships between people. We generate a set of daily networks by way of bond percolation, where each link of the underlay becomes a member of the daily network with probability \( p \).

Nodes with no remaining links in the daily networks are removed, and so the daily networks tend to be smaller than the underlay when \( p \) is small:

However, as you can see, the overlap is smaller than one would expect due to probability alone, except for very small values of \( p \).

**Central Question**
To what extent do these daily networks exhibit dynamic centrality?

![Graph showing bond percolation](image)

**Hub Overlap**
One way to measure dynamic centrality is to list the hubs in two daily networks, and calculate the fraction of nodes that appear in both lists. Here we calculate this overlap for pairs of daily networks built from an underlay of 100k nodes. A hub is a node ranked* in the top 1% or 10% of its network.

The overlap naturally approaches zero along with the size of the daily network: probabilistically, any hub in one daily network has only a \( N_d/N_U \) chance of being in the other network, let alone being a hub, and so the overlap should be proportional to this ratio.

![Graph showing overlap](image)

However, as you can see, the overlap is smaller than one would expect due to probability alone, except for very small values of \( p \).

**Underlay–Daily Correlations**
We can also look at the rank that a node has in the daily networks, as a function of its rank in the underlay. To do this, we generate 500 daily networks from the same underlay, and count the number of times each node has a daily rank in the following categories:

*Top 1%* 1–3% 3–10% 10–30% 30%+

As \( p \) becomes larger, nodes become less likely to leave their category. We visualize this in the following graphs, which show the fraction of times that nodes in a particular underlay category appear in each daily category.

**References**
1. This term was first introduced by D. Braha and Y. Bar-Yam in *Complexity* 12(2):59-63 (2006).