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Numerical analysis of a time-headway bus route model

Scott A. Hill

Department of Physics, Northeastern University, Boston, MA 02115, USA

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Abstract

In this paper, we consider a time-headway model, introduced in Ref. [Physica A 296 (2001) 320], for buses on a bus route. By including a simple no-passing rule, we are able to enumerate and study the unstable modes of a homogeneous system. We then discuss the application of the model to realistic scenarios, showing that the range of reasonable parameter values is more restricted than one might imagine. We end by showing that strict stability in a homogeneous bus route requires careful monitoring by each bus of the bus in front of it, but in many cases this is unnecessary because the time it takes for the instability to appear is longer than a bus would normally spend on a route.

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1. Introduction

While there has been much interest in the study of automobile traffic (for a review, see Ref. [1]), there have been few corresponding studies of buses [2–5]. The dynamics of a bus route, while having some similarities with that of automobile traffic, differs due to the added interaction of buses with passengers at designated bus stops. A good reason for studying the dynamics of bus routes is that they are so often unstable. Buses are initially spaced at regular intervals. However, if one bus is delayed for some reason, it will find a larger number of passengers waiting for it at subsequent stops, delaying it further. Meanwhile, the bus following finds fewer passengers waiting for it,

E-mail address: sahill@mailaps.org (S.A. Hill).

allowing it to go faster until eventually it meets up with the delayed bus. Clusters of three, four, or more buses have been known to form in this manner, resulting in slower service.

In Refs. [3,4], Nagatani presents a time-headway model for buses. Using linear stability analysis, he is able to determine the range of parameters over which the homogeneous solution (i.e., with buses spaced evenly apart) is unstable. In this paper, we make a more thorough investigation of Nagatani’s model. We demonstrate the existence of three types of phase diagrams, in which the behavior of the bus system is divided into four separate categories. We conclude with a discussion of how this model may be applied to real-world situations, and the limitations imposed by practical considerations.

2. Model

We consider the following model, introduced in Ref. [4], of buses on a bus route (Fig. 1). Bus stops are labelled by $s = 1, 2, \dots$ where stops s and $s + 1$ are a distance L apart. There are J buses, $j = 1, \dots, J$, which travel from stop to stop, with bus $j = 1$ in the lead and bus $j = J$ in the rear. Every bus visits every stop, and buses do not pass one another. The time at which bus j arrives at stop s is $t_{j,s}$, which is given by the recursive relation

$$t_{j,s} - t_{j,s-1} = \lambda\gamma\delta t_{j,s-1} + \frac{L}{V_{j,s-1}}, \tag{1}$$

where

$$\delta t_{j,s} \equiv t_{j,s} - t_{j-1,s} \tag{2}$$

is the *time-headway*, the time gap in front of bus j at stop s . The first term on the right-hand side of Eq. (1) is the time it takes for passengers to board the bus at stop $s - 1$. The parameter λ is the rate at which passengers arrive at a bus stop; $\lambda\delta t_{j,s-1}$ is the number of passengers that have arrived at stop $s - 1$ since the previous bus left. The parameter γ is the time it takes for each passenger to board the bus, so $\lambda\gamma\delta t_{j,s-1}$ is the amount of time needed to board all of the passengers. For convenience, we introduce

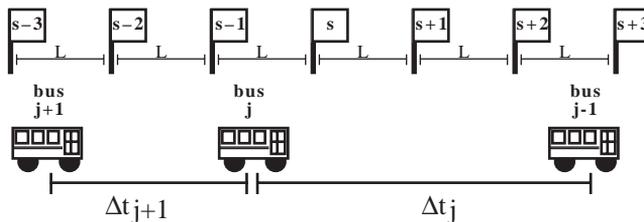


Fig. 1. Schematic illustration of the model.

the dimensionless parameter $\mu \equiv \lambda\gamma$, which we call the *passenger rate*. For simplicity we ignore the passengers getting off the bus; we will assume that it takes less time for the passengers to get off than it does to get on and pay the fare.

The second term in Eq. (1) is the time it takes for bus j to travel from stop $s-1$ to stop s , where $V_{j,s-1}$ is the average velocity of the bus between stops. If this velocity is constant, then the tendency for buses to bunch together, as described in the introduction, has no counterweight, and a steady flow of buses will always be unstable (unless there are no passengers). It is reasonable to assume, however, that a bus driver will try to prevent bunching by slowing down when the gap between his bus and the next is too small. One can model this by writing the average speed $V_{j,s}$ as a function $\tilde{V}(\delta t_{j,s})$ of the gap between his bus and the bus in front of him:

$$\tilde{V}(\delta t) = v_{\min} + (v_{\max} - v_{\min}) \frac{\tanh \omega(\delta t - t_c) + \tanh \omega t_c}{1 + \tanh \omega t_c}. \quad (3)$$

The hyperbolic tangent factor acts as a spread-out step function, centered at t_c with a width proportional to $1/\omega$. The parameter v_{\max} is the speed a free bus (i.e., one that is alone on the route) would travel. On the other hand, v_{\min} is the speed a bus travels if it has completely caught up with the bus in front of it. For example, if $v_{\min} = 0$ then a bus which has caught up with the bus in front of it will stop and wait until its predecessor has cleared the next stop before proceeding.

In what follows, it is convenient to work with the time headways $\delta t_{j,s}$, rather than the arrival times $t_{j,s}$. It is also convenient to rewrite our expressions in terms of dimensionless quantities. In doing so, we find that there are four significant parameters, not including initial conditions. The first such parameter is the passenger rate μ . The other three are $\alpha = L\omega/v_{\max}$, $\beta = v_{\min}/v_{\max}$, and $\varepsilon = 1 - \tanh \omega t_c$ (which will typically be small). We will also consider the dimensionless variable $\Delta t_{j,s} = \omega \delta t_{j,s}$ and the dimensionless velocity function $V(\Delta t) = \tilde{V}(\Delta t)/v_{\max}$. Our evolution equation Eq. (1) now reads

$$\Delta t_{j,s} - \Delta t_{j,s-1} = \alpha \left[\frac{1}{V(\Delta t_{j,s-1})} - \frac{1}{V(\Delta t_{j-1,s-1})} \right] + \mu [\Delta t_{j,s-1} - \Delta t_{j-1,s-1}], \quad (4)$$

where

$$V(\Delta t) = \beta + \frac{(1 - \beta)\varepsilon \tanh \Delta t}{1 - (1 - \varepsilon) \tanh \Delta t} = \frac{\beta(1 - \tanh \Delta t) + \varepsilon \tanh \Delta t}{(1 - \tanh \Delta t) + \varepsilon \tanh \Delta t}. \quad (5)$$

3. Stability analysis

We are interested in the stability of a homogeneous flow of buses, with $\Delta t_{j,s} = \Delta t_{j,0} = \Delta t_0$. One can easily verify that this is a solution to Eq. (4). We introduce a small perturbation to the initial homogeneous solution: $\Delta t_{j,s} = \Delta t_0 + y_{j,s}$, where $y_{j,s}$

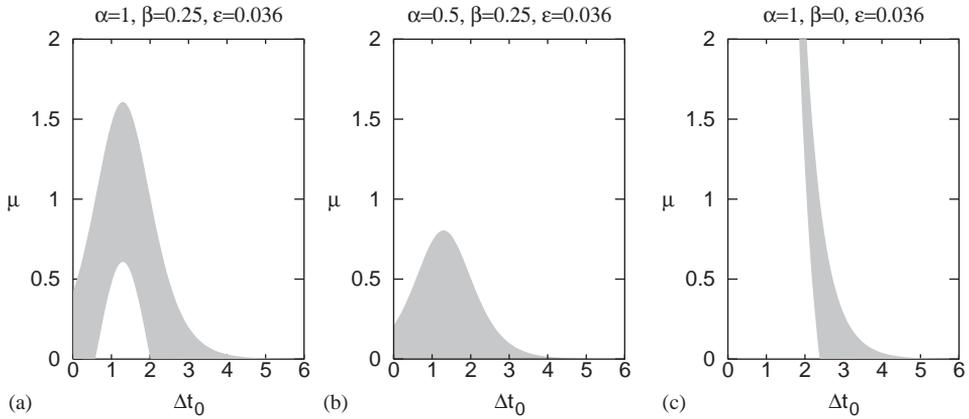


Fig. 2. (a–c) Phase diagrams indicating the regions of stability of Eq. (6), for three representative values of the parameters. The shaded area is the region which satisfies Eq. (8).

is small. To first order, Eq. (4) becomes

$$y_{j,s} - y_{j,s-1} = [y_{j+1,s-1} - y_{j,s-1}][F(\Delta t_0) - \mu], \tag{6}$$

where we have introduced the convenient abbreviation

$$F(\Delta t_0) \equiv \alpha \frac{V'(\Delta t_0)}{V(\Delta t_0)^2} = \frac{\alpha(1 - \beta)\epsilon(1 - \tanh^2 \Delta t_0)}{[\beta(1 - \tanh \Delta t_0) + \epsilon \tanh \Delta t_0]^2}. \tag{7}$$

It can be shown [4] that the perturbation is stable if

$$F(\Delta t_0) - 1 < \mu < F(\Delta t_0). \tag{8}$$

From Eq. (8) we can construct a phase diagram (Fig. 2) for the stability of an initially homogeneous bus route, based on the initial spacing Δt_0 and the passenger rate μ . The stable region in phase space is bounded by the curves $\mu = F(\Delta t_0)$ and $\mu = F(\Delta t_0) - 1$. Because of the added constraint that $\mu \geq 0$, there are different phase diagrams depending on whether $F(\Delta t_0) - 1$ is ever positive (Fig. 2a) or not (Fig. 2b). The curve $F(\Delta t)$ has a maximum value of

$$\alpha \frac{(1 - \beta)}{2\beta - \epsilon} \quad \text{at } x = 1 - \frac{\epsilon}{\beta}, \tag{9}$$

so the phase diagram resembles Fig. 2a whenever

$$\alpha > \frac{2\beta - \epsilon}{1 - \beta} \approx \frac{2\beta}{1 - \beta}. \tag{10}$$

A third phase diagram, Fig. 2c, arises when $v_{\min} = 0$, as it is in Fig. 3 of Ref. [4] (although apparently not in Fig. 8 of the same reference, which may account for the discrepancy between those two phase diagrams).

4. Simulation

To study the ways in which the system becomes unstable, we evaluate Eq. (4) iteratively in s . Our initial condition is

$$\Delta t_{j,0} = \Delta t_0 + 0.1r_j, \quad (11)$$

where r_j are random numbers chosen between -1 and 1 . For each combination of initial headway Δt_0 and passenger rate μ , we run the simulation until either (a) we reach stop $s = 5000$, or (b) one or more of the bus headways exceeds $\Delta t = 1000$ (in which case the system has become unphysical).

In this paper we consider two different boundary conditions. The first is periodic in the bus number j ; so for example $\Delta t_{1,s} = t_{1,s} - t_{J,s}$. This is convenient numerically, and it creates translational symmetry, but it is hard to construct a physical model which begins with this characteristic. We also consider a fixed boundary condition, where $\Delta t_{1,s} = \Delta t_0$. Since the velocity of a bus depends entirely on Δt , this corresponds to a scenario where the initial bus ($j = 1$) moves at a constant speed $V(\Delta t_0)$.¹

The structure of the model requires that buses not pass one another; however, there is nothing in Eq. (4) to prevent the headways Δt from becoming negative. To fix this, we add to our simulation the rule that any $\Delta t_{j,s} < 0$ is replaced by $\Delta t_{j,s} = 0$. This corresponds to a situation where drivers are forbidden (or unable due to road conditions) to pass one another.²

Fig. 3 shows the results of our simulation runs for a typical set of parameters ($\alpha = 1$, $\beta = \frac{1}{4}$, $\varepsilon = 1 - \tanh 2 = 0.036$), using both types of boundary conditions. In both cases, the phase space is divided into four regions, corresponding to four types of runs.

4.1. Stable runs

Most of the runs within the stable region, as defined by Eq. (8), remain homogeneous. In the periodic case, the initial fluctuations in Δt_j settle into a small precessing sinusoidal perturbation which decays exponentially with time (Fig. 4). In the fixed case, the system quickly locks onto the constant solution $\Delta t = \Delta t_0$ with no fluctuations.

4.2. Explosive runs

Most of the runs lying above the stable region quickly develop an unphysical instability. This takes the form seen in Fig. 5, independent of boundary condition: those headways lying above the mean increase exponentially, while those lying below

¹ A more realistic boundary condition may involve ramping up the passenger rate μ over time, as occurs during the course of a normal day.

² An alternative solution which we do not consider here is to allow buses to pass one another. This could conceivably be done by replacing all $\Delta t_j < 0$ with $|\Delta t_j|$, effectively swapping the labels of buses that pass one another. We have not considered whether this would work in practice, however.

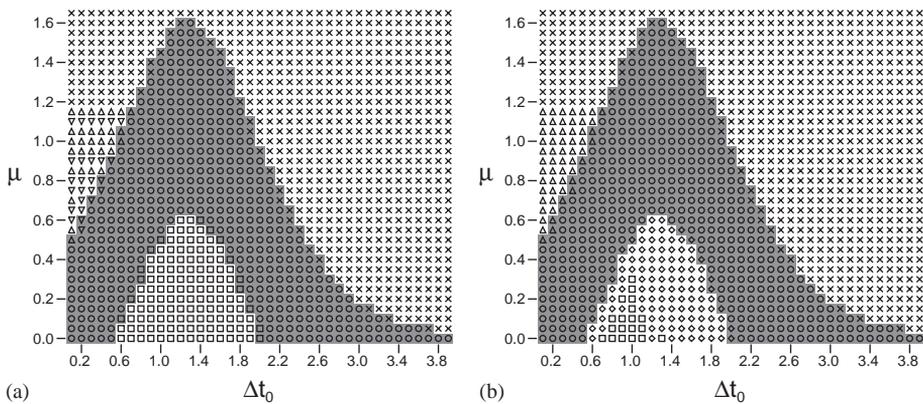


Fig. 3. Phase diagram for bus systems with (a) periodic and (b) fixed boundary conditions, where $\alpha = 1$, $\beta = \frac{1}{4}$, and $\varepsilon = 0.036$. The horizontal axis is the initial time headway Δt_0 , while the vertical axis is the passenger rate μ . Stable runs (Section 4.1) are marked by circles (\circ) and exploding runs (Section 4.2) by exes (\times). Oscillatory solutions (Section 4.4) are marked by squares (\square); diamonds (\diamond) mark runs which started like oscillatory solutions but ended up flat. Slowed solutions (Section 4.3) with clusters are marked by upward-pointing triangles (\triangle) and slowed solutions without clusters by downward-pointing triangles (∇). The gray shading shows the region where $F(\Delta t_0) - 1 < \mu < F(\Delta t_0)$.

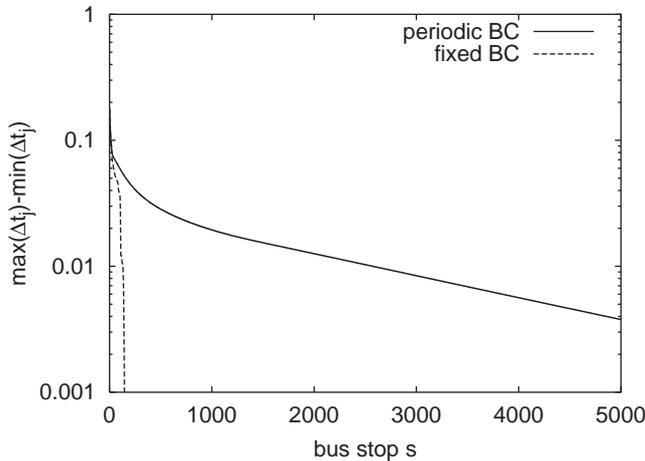


Fig. 4. A plot of $\max(\Delta t_j) - \min(\Delta t_j)$ versus iteration step s shows that the periodic system ($\mu = 0.8$, $\Delta t_0 = 1.5$) is converging exponentially to the homogeneous solution. The same system with the fixed boundary condition converges much more quickly.

decrease steadily until they reach zero. An observer stationed at a stop far down the line will see clusters of buses arriving after long waits; far enough down the line, these waits become astronomical, which is absurd. Clearly this model is insufficient to deal with these runs at long times.

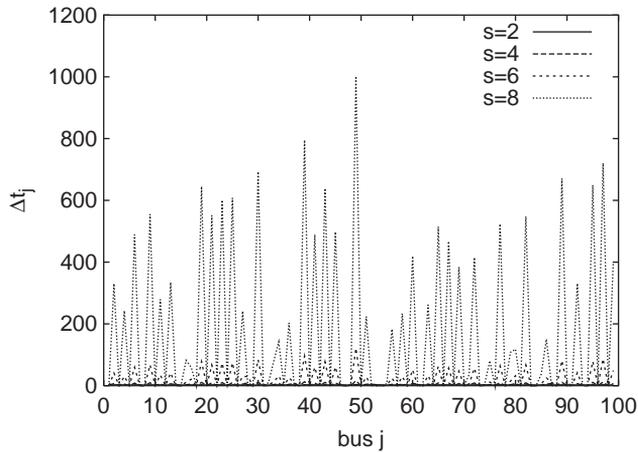


Fig. 5. An extreme example of an explosive run, with $\mu = 1.9$ and $\Delta t_0 = 2.5$. By stop $s = 8$ there are buses which are already 1000 time units apart (where one time unit is the time it takes for a free bus to travel from one stop to the next).

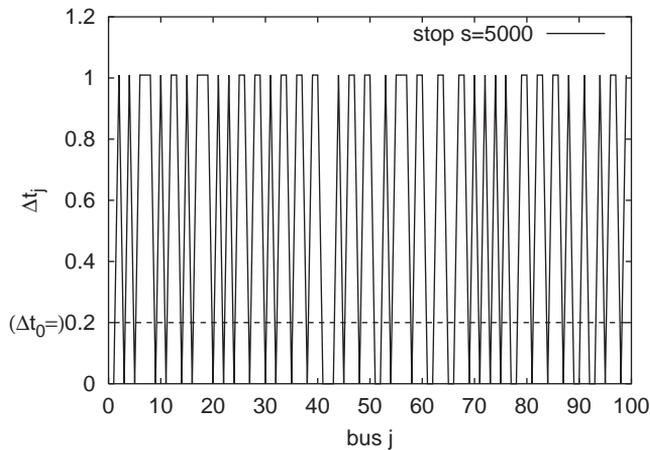


Fig. 6. An example of a slowed run, where $\mu = 0.95$ and $\Delta t_0 = 0.2$.

4.3. Slowed runs

To the left of the stable region are runs which develop an alternative stable solution, as seen in Fig. 6. In the case of the fixed boundary condition, these runs have two things in common. The first, indicated by the vanishing of one or more headways, is the appearance of clusters: two or more buses which travel along as a single unit. The second is that the units, whether single buses or clusters, are homogeneously spaced, but with a spacing that is larger than the initial spacing Δt_0 . It should be pointed out

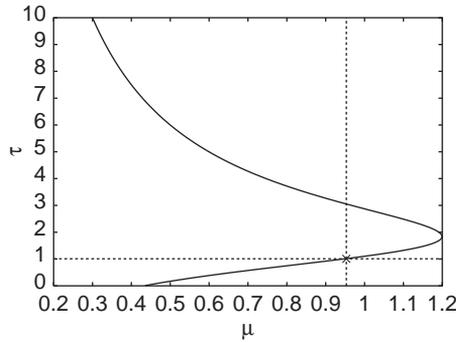


Fig. 7. The solution to Eq. (13) for the set of parameters $\alpha = 1$, $\beta = \frac{1}{4}$, $\varepsilon = 0.036$. The \times marks the solution corresponding to Fig. 6. In simulation, the spacing τ between slowed buses always comes from the lower branch of the curve.

that the solutions shown here are stationary; the clusters and spacings, after they form, do not change.

In analytic terms, these states are of the form $\Delta t_{j,s} = \Delta t_j = \tau r_j$, where r_j is either 0 or 1, and $\tau > \Delta t_0$ is a constant. It is straightforward to show that this is a solution to Eq. (4):

$$0 = \alpha \left[\frac{1}{V(\tau r_j)} - \frac{1}{V(\tau r_{j-1})} \right] + \mu \tau [r_j - r_{j-1}]. \tag{12}$$

When $r_j = r_{j-1}$, this equation is satisfied trivially. Otherwise, the equation takes the form

$$\mu = \frac{\alpha}{\tau} \left(\frac{1}{\beta} - \frac{1}{V(\tau)} \right), \tag{13}$$

which we can solve numerically for τ (Fig. 7). For a given passenger rate μ , these spacings τ correspond precisely with those seen in simulation. Furthermore, for high enough passenger rates— $\mu > 1.199$ for this set of parameters—Eq. (13) has no real solutions, which explains the cut-off in Fig. 3 between the slowed and explosive regimes.

In the case of the periodic boundary condition there are cases where the clusters eventually break up, leaving a system of buses which are equally spaced, but with the larger spacing predicted by Eq. (13). These runs are marked by downward-pointing triangles (∇) in Fig. 3.

4.4. Oscillatory runs

In the case of a run lying below the stability region in phase space, the first term in Eq. (4), which is meant to resist the tendency for buses to cluster, becomes too large. This leads to overreaction, so that two buses which arrive too close together at one stop are too far apart at the next. The resulting behavior may be compared to a system of underdamped oscillators. Fig. 8 shows the resulting behavior. For the periodic boundary condition, these oscillations decay as a power law (Fig. 9), but at

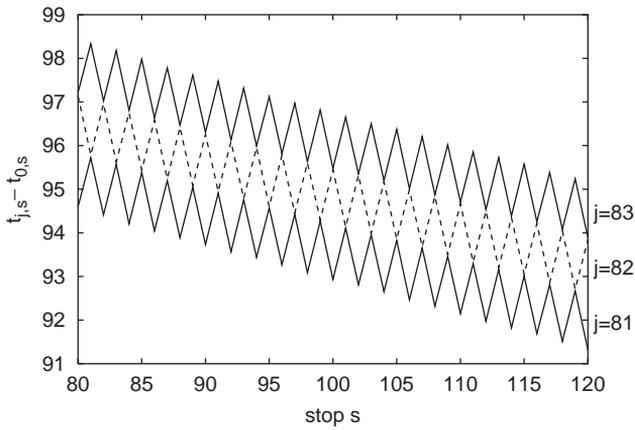


Fig. 8. An example of the oscillatory condition, with $\mu = 0.1$, $\Delta t_0 = 1.0$, and fixed boundary conditions. This plot shows how long each of three consecutive buses arrive at a stop s after the initial bus $j = 1$ arrived. Notice that the middle bus is bunched first with the bus preceding it, then the bus following, and so forth.

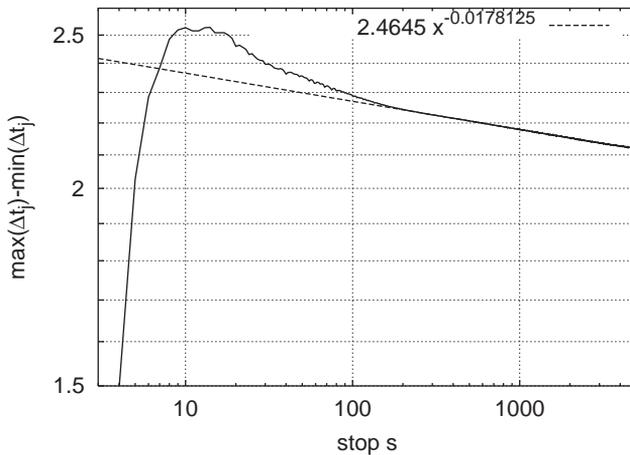


Fig. 9. The decay of an oscillatory run with periodic boundary conditions, $\mu = 0.2$, $\Delta t_0 = 1.2$. Both axes are logarithmic. The power-law decay is too small to have an appreciable effect on the behavior of the buses.

so slow a rate as to be practically permanent. With the fixed boundary condition, the earliest buses (i.e., those with the lowest j) shed the oscillating behavior after only a few stops, resuming a homogeneous configuration; with more iterations, more buses join the homogeneous regime. In some cases, such as in Fig. 10, the system reaches a steady state with the oscillations still dominating the later buses. In other cases, however, the system becomes completely homogeneous, as the bifurcation point seen in Fig. 10 slips off the right-hand side of the graph. This later effect may be due to the finite number of buses.

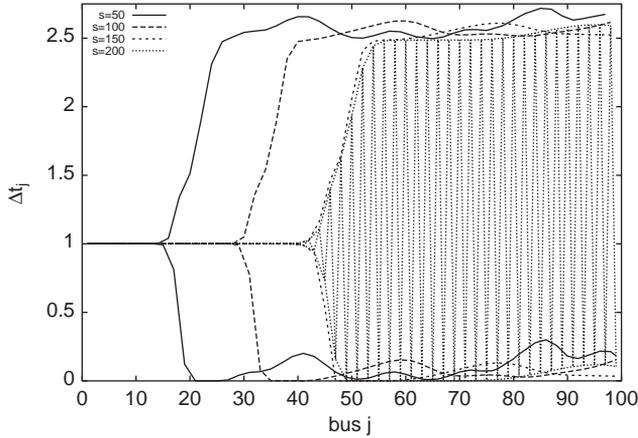


Fig. 10. The headways of an oscillatory system with fixed boundary conditions, $\mu = 0.1$, $\Delta t_0 = 1.0$, at four different bus stops. The bifurcation of each line indicates the presence of oscillatory behavior, which we show in full for the $s = 200$ case.

5. Discussion

In our simulations we have considered a large range of values for μ and Δt_0 . However, these parameters should be limited by a couple of practical concerns.

The passenger rate μ is defined as the product of the number of passengers that arrive at a stop per unit time, and the time it takes a single person to board the bus. Put another way, it is the ratio of the number of people that arrive at a stop to the number of people who can board the bus in the same amount of time. This number must be less than one; if not, then passengers arrive at a stop faster than the bus can take them on, and the bus should never be able to leave the stop. Since $0 < \mu < 1$, only one of the two inequalities in Eq. (8) is meaningful for a given value of the parameters (including Δt_0), depending on whether $F(\Delta t_0)$ is larger or smaller than 1. This suggests that if one wanted to maximize the area of the stability region in phase space, one would do well to make sure that the lower stability curve $F(\Delta t_0) - 1$ just grazes zero, or that $\alpha(1 - \beta) = 2\beta - \varepsilon$ according to Eq. (9).

Another practical consideration puts a limit on the value of Δt_0 . Typically, buses are spaced far enough apart so that the first bus will reach the first stop before the second bus is allowed to leave, particularly if the stops are spaced fairly close together. This is described by the inequality

$$\Delta t_0 > \frac{L}{\tilde{V}(\Delta t_0)} = \alpha \frac{(1 - \tanh \Delta t_0) + \varepsilon \tanh \Delta t_0}{\beta(1 - \tanh \Delta t_0) + \varepsilon \tanh \Delta t_0} > \alpha . \tag{14}$$

For the parameters we have been studying,

$$\Delta t_0 > \frac{1 - 0.964 \tanh \Delta t_0}{0.25 - 0.214 \tanh \Delta t_0} \Rightarrow \Delta t_0 > 1.82 . \tag{15}$$

This cuts out much of the interesting part of Fig. 3, including the slowed runs and almost all of the underdamped solutions. In our discrete model, the basic iteration step is the bus stop; bus drivers are allowed to change their speed at the bus stops and nowhere else. If the buses are several stops apart, then they have enough time to react to one another. Otherwise, unusual situations such as the slowing case or the underdamped case may arise.

Finally, we consider the relationship between δt_0 and t_c , which is how close a bus will come to the bus in front of it without slowing down. Consider the ratio

$$r = \frac{1 - \tanh \Delta t_0}{\varepsilon} = \frac{1 - \tanh \omega \delta t_0}{1 - \tanh \omega t_c}. \quad (16)$$

We can rewrite $F(\Delta t_0)$ in terms of this ratio:

$$F(\Delta t_0) = \alpha(1 - \beta) \frac{r(2 - \varepsilon r)}{[\beta r + 1 - \varepsilon r]^2}. \quad (17)$$

If $\varepsilon r = 1 - \tanh \Delta t_0 \ll 1$ (which it will be if Eq. (15) is valid, since $1 - \tanh 1.82 = 0.05$), then

$$F(\Delta t_0) \approx \alpha(1 - \beta) \frac{2r}{(1 + \beta r)^2}. \quad (18)$$

Now let us consider what values our parameters might take in real life. A typical urban bus route might have $L = 0.5$ km, $v_{\max} = 50$ km/h = $\frac{5}{6}$ km/min, $v_{\min} = 15$ km/h = $\frac{1}{4}$ km/min, and $\omega = 1$ /min; thus $\alpha = 0.6$ and $\beta = 0.3$. A bus which runs every 10 min might take on two passengers at every stop, so $\lambda = 0.2$ people per minute. If it takes $\gamma = 3$ s for a person to board a bus, then $\mu = \lambda\gamma = 0.01$.

Consider a scenario where bus drivers only react to what they see; that is, a driver will only slow down if she can see the next bus in front of her. It takes a free bus $L/v_{\max} = 0.6$ min to travel from one stop to the next, so a reasonable value for the amount of warning a bus driver has is on the order of $t_c = 1$ min. Typical bus routes have buses which are spaced much farther apart, perhaps every $\Delta t_0 = 10$ min or more. In this scenario, $r = (1 - \tanh 10)/(1 - \tanh 1) = 10^{-8}$, so $F(\Delta t_0) \approx 10^{-8}$. Since $\mu = 10^{-2}$, the stability condition in Eq. (8) is very clearly violated. For $F(\Delta t_0)$ to reach a high enough value to create stability, we need in general for the ratio r to be closer to 1. $F(r)$ takes its maximum value when $r = 1/(2\beta)$, in which case $F(r) = 1/(8\beta) = 0.4$, which is easily larger than μ in this example. Notice that, for values of t_c and δt_0 greater than 1 min, $r \approx e^{2(t_c - \delta t_0)}$, so for each minute's difference between t_c and δt_0 , r is increased or decreased by a factor of 10. It would seem that to maintain a stable homogeneous bus route, drivers must be reacting to the bus in front of them even from the very beginning, and can only ignore the leading bus if they have gotten far enough behind (in which case, of course, the proper solution is to go as quickly as possible).

This is quite a stringent requirement for stability, and explains why it is so common to see clusters of buses in large cities. It does not seem likely, however, that this would be the case for less frequent bus service, such as when buses run once per hour. A driver on such a route does not typically keep track of what the previous bus was doing an hour ago, and yet one does not see clustering behavior on these low-frequency routes. The reason for this is that the instabilities predicted by Eq. (8) may take a long

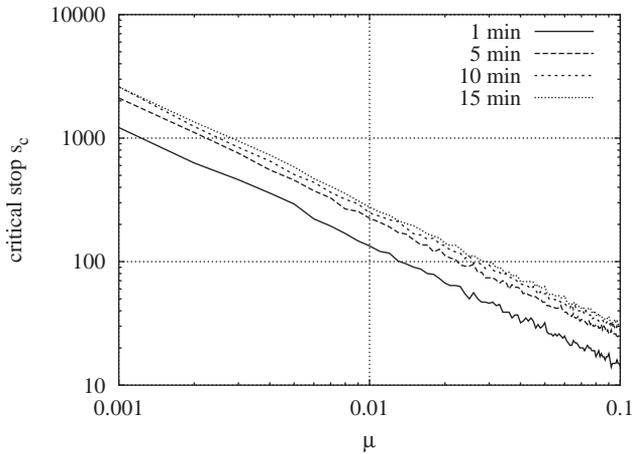


Fig. 11. This shows the number of stops, for a given passenger rate μ , that the bus route will run before seeing a 1-min, 5-min, 10-min, or 15-min deviation in the initial homogeneous state. The parameters used in this simulation are $\alpha = 0.6$, $\beta = 0.3$, $\varepsilon = 1 - \tanh 1$, and $\delta t_0 = 60$ min. A power-law fit to all four lines shows that they all go as $\mu^{-0.965 \pm 0.005}$.

time to become noticeable, and normal bus routes tend to have a limited number of stops. Fig. 11 shows the number of stops that a bus route has to cover before seeing a noticeable deviation in the initial homogeneous state. If the passenger rate is $\mu = 0.01$ as suggested in the urban case above, then the route would have to have 130 stops to show a 1 min deviation from the homogeneous state, and 225 stops to show a 5 min deviation. If $L = 0.5$ km, these correspond to 65 and 112 km, longer than your average bus route. The situation is even better when you consider that a suburban or rural bus route might have, not 1 person for every 5 min, but maybe 1 person every 25 min, so that $\mu = 0.002$, and we can start having bus routes with 600 or 1000 stops before the instability becomes noticeable. This is not to say that smaller routes remain perfectly on time, of course; just that the delays are unlikely to be due to the need to pick up extra passengers. Since buses will typically complete the route only to turn around and do it again, one might consider an entire day's run to be a single route, in which case instabilities may creep in late in the day. However, the introduction of a bus terminal, where buses wait until a specific time to leave for their next trip through the route, would have to be accounted for in this case.

In this paper, we have considered the bus route model proposed in Ref. [4]. We have added a simple way to deal with negative time-headways (by replacing all negative Δt_j 's with zeroes), and by doing so have been able to study the unstable modes of a homogeneous system of buses. We show that there are in fact three different phase diagrams (Fig. 2) for this system, depending on our choice of parameters, and that in addition to the stable homogeneous state, there are three unstable modes which the system can fall into: the explosive mode, the slowed mode, and the underdamped mode. We then considered the application of this model to real-life bus routes. We have shown that the passenger rate μ and the initial spacing between buses Δt_0 are

greatly restricted by practical considerations, and that to guarantee stability one needs to have bus drivers who are constantly tracking the bus in front of them, even when that bus is at its normal distance. Fortunately, this is only necessary for bus routes with very many stops; with fewer stops to make, a bus may be able to complete the route before any instabilities can become noticeable.

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